

# The Liar Paradox 1

PHIL2511 Paradoxes

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Seminar 8

4 April 2013

# Admin

Required reading:

Sainsbury, Chapter 6, Sections 6.1-6.6

Reading for next week:

i) Sainsbury, Chapter 6, Sections 6.7-6.9

ii) Goldstein, 'This statement is not true' is not true

Essay 2 questions: Will be handed out next week

Seminar 9: Liar Paradox 2 (12 April)

Seminar 10: Dialethism (19 April, Last seminar)

# The Liar

(L1) L1 is false

Rough version of the paradoxical argument:

Suppose L1 is true. Then, since L1 says that L1 is false, it follows that L1 is false.

Suppose, instead, that L1 is false. Then it is not the case that L1 is false, and hence L1 is true.

Hence, in either case, we get a contradiction!

# Premises of the more formal version of the paradoxical argument

(Bivalence) Every sentence is either true or false

(No gluts) No sentence is both true and false

(T)  $S$  is true iff  $p$ , where  $S$  says that  $p$

# Instances of (T)

- (1) 'Grass is green' is true iff grass is green, since 'Grass is green' says that grass is green
- (2) 'Le neige est blanch' is true iff snow is white, since 'Le neige est blanch' says that snow is white

# The paradoxical argument

P1) Either L1 is true or L1 is false [By (Bivalence)]

Suppose L1 is true.

Then: by (No Gluts) it is not the case that L1 is false

Since, L1 says that L1 is false, it follows from (T) that L1 is not true

So we have:

P2) If L1 is true, then L1 is both true and not true.

# The paradoxical argument (cont)

Suppose instead that L1 is false.

Then: since L1 is false, it follows from (T) that L1 is true.

By (No Gluts), since L1 is false, L1 is not true.

So we have:

P3) If L1 is false, then L1 is both true and not true.

(P4) then follows from (P1-3), which is a contradiction.

(P4) L1 is both true and not true.

# Solution 1

(Bivalence) is false, and (G) is true.

(G) L1 is neither true nor false

Challenge facing this response: Need to give an independent argument for (G)



# Plausible examples of sentences which are neither true nor false

(3) When did the dinosaurs become extinct?

(4) You have stopped beating your wife

(5) That elephant is going to charge

# A modified version of (Bivalence)

(MB) If  $S$  is a non-defective representation of how the world is or fails to be, then  $S$  is either true or false

(3-5) aren't counterexamples to (MB), since (1-3) aren't non-defective representations of how the world is or fails to be

Given (MB), proponents of solution 1 must claim that L1 fails to be a non-defective representation of how the world is or fails to be

Challenge to proponents of solution 1: Explain why this is so

# Grounding

An attractive idea is the following:

(Grounding)  $S$  non-defectively represents how the world is or fails to be iff  $S$  is grounded in how the world is, where how the world is can be described without employing the concept of truth

# The idea of grounding illustrated

Suppose Jane knows most of English, but doesn't know the meaning of 'truth'.

Q: How could we explain the meaning of 'truth' to her?

A: We could give her the following recipe

“You should call sentence **true** iff you are willing to assert it.

You should call a sentence **false** iff you are willing to assert it's negation”

# The idea of grounding illustrated (cont)

Using this recipe, Jane is able to understand the following sentences:

(6) 'Snow is white' is true

(7) 'Grass is green' is true

Using this understanding, she can then use the above recipe to understand the following sentences:

(6a) "'Snow is white' is true' is true

(7a) "'Grass is green' is true' is true

# The idea of grounding illustrated (cont)

However, Jane will never be able to use the recipe and the understanding she has obtained to understand the liar sentence L1.

(L1) L1 is false

Reason: In order to use the recipe to understand (L1), Jane would need to already understand (L1)! Hence the recipe can't be used to work out the meaning of (L1)

Claim: The meaning of 'true' in English is just what it would be if it was introduced by the above recipe. The meaning of (L1) is therefore not defined, and which is why (G) is true.

# Other sentences that fail to be grounded

(T1) T1 is true

(S1) S2 is true

(S2) S3 is true

(S3) S1 is true

According to proponents of solution 1, these sentences are neither true nor false, since they fail to be grounded in how the world is

# Problem: Solution 1 doesn't work for the strengthened liar

(L2) L2 is not true

The paradoxical argument:

Q1) Either L2 is true or L2 is not true

Suppose L2 is true.

Then: by (T) since L2 says that L2 is not true, L2 is not true.

So we have:

Q2) If L2 is true, then L2 is both true and not true



# The strengthened liar (cont)

Suppose, instead, that L2 is not true.

Then: by (T), since L2 says that L2 is not true, it is not the case that L2 is not true. Hence, L2 is true.

So we have:

Q3) If L2 is not true, then L2 is both true and not true

(Q4) then follows from (Q1-3), which is a contradiction.

Q4) L2 is true and not true

# $(G^*)$ can't help solve the strengthened liar

- Let  $(G^*)$  be  
 $(G^*)$  L2 is neither true nor false
- (Bivalence) wasn't one of the premises of the above argument (the only premises were (T) and (No-Gluts)). So  $(G^*)$  doesn't invalidate the argument.
- $(G^*)$  entails that L2 is not true, and hence entails the paradoxical L2. Anyone committed to  $(G^*)$  is therefore committed to L2

# Tarski's solution

- Our ordinary concept of truth is incoherent
- Similarly 'true' in English is incoherent, or badly defined
- For scientific and other theoretical purposes we should stop using 'true', and define replacement notions—truth in L, for different languages L—that can do the theoretic work 'truth' does in English
- For any well-defined language L, we can define a truth predicate for L 'True-in-L' which satisfies Tarski's test of adequacy for a truth predicate for L so that if S is a sentence in L, S means that p, then: S is true-in-L iff p.

# Tarski's solution (cont)

- We can make sure we avoid the liar paradox by requiring that this predicate 'true-in-L' is not among the expressions of L
- In order to study truth-in-L, we can then use a language other than L which does contain the 'true-in-L' predicate
- Upshot of Tarski's solution:
  - i) Our ordinary notion of truth is incoherent, but there are language-relative notions of truth that are coherent.
  - ii) We can study 'true-in-L' where L is a different language from that we are using, but we can't study 'true-in-L' using L itself.

# Tarski's replacements illustrated

Suppose Lang1 is a language containing no semantic notions (so that, for example, it doesn't contain 'true' or any names for linguistic expressions in Lang1)

Let Lang2 be a language containing all the sentences Lang1, together with

- i) For any sentence S in Lang1, 'S' is a name in Lang2 referring to S
- ii) The predicate 'True-in-Lang1' in Lang2 having the following definition: a) For any sentence S in Lang 1, we should be willing to endorse "'S' is true-in-Lang1' iff we are willing to endorse S; and b) For any x, that is not a sentence in Lang1, we should deny 'n is true-in-Lang1', where n is a name for x

# Tarski's replacements illustrated (cont)

Let Lang3 be a language containing all the sentences Lang2, together with

- i) For any sentence  $S$  in Lang 2, ' $S$ ' is a name in Lang3 referring to  $S$
- ii) The predicate 'True-in-Lang2' in Lang3 having the following definition: a) For any sentence  $S$  in Lang 2, we should be willing to endorse " $S$  is true-in-Lang1' iff we are willing to endorse  $S$ ; and b) For any  $x$ , that is not a sentence in Lang2, we should deny ' $n$  is true-in-Lang1', where  $n$  is a name for  $x$

# Why 'true-in-Lang1' does not give rise to paradox when it is in Lang2

Q: Does  $(S^*)$  give rise to paradox in Lang2?

$(S^*)$   $S^*$  is not true-in-Lang1

A: No, since

- i)  $(S^*)$  is not a sentence in Lang 1, so there is no name in Lang2 referring to it. Hence, ' $S^*$ ' is not a name in Lang2; and
- ii) Even if ' $S^*$ ' was a name in Lang2, there is no problem with endorsing  $(S^*)$  since we can't use the fact that  $(S^*)$  satisfies Tarski's test to get a contradiction.

# Objections to Tarski's solution

- Tarski's solution is too radical. According to Tarski (6) is incoherent, since 'true' is incoherent'. But surely (6) is coherent. Indeed, surely (6) expresses a way the world is, and hence is true!

(6) 'Snow is white' is true

- There are ways things are, and ways things fail to be. We can give the following definition of truth: S is true iff S expresses a way things are. Doesn't this define a coherent notion of truth? If not, why not?