# Zeno's Paradoxes 

PHIL2511 Paradoxes
Dan Marshall
Seminar 3
8 February 2013

## Admin

Required reading for this seminar: Sainsbury,
Chapter 1
Required reading for next seminar: Sainsbury, Chapter 2, Sections 2.1-2.2
Essay 1 due: Thursday March 7, 5pm (Hand in to Philosophy Office)

## Zeno

- Zeno was an Ancient Greek Philosopher born around 490BC
- He lived in the town of Elea (now in Southern Italy)
- None of his writings have survived
- All we know of him is from what other Greek philosophers (such as Plato and Aristotle) wrote about him


## Zeno's Paradoxes

Zeno produced a number of paradoxes, most of which have been lost.
The most famous are:
i) The Racetrack
ii) Achilles and the Tortoise
iii) The Arrow

## The Racetrack

A runner starts from Z and ends at $\mathrm{Z}^{*}$

Call $\mathrm{Z1}$ the midpoint between Z and $\mathrm{Z}^{*}$
Call $\mathrm{Z2}$ the midpoint between $\mathrm{Z1}$ and $\mathrm{Z}^{*}$
Call Z 3 the midpoint between $\mathrm{Z2}$ and $\mathrm{Z}^{*}$ and so on

## The Racetrack (cont)

(1) Going from $Z$ to $Z^{*}$ would require one to complete infinitely many journeys: Z to Z1, Z1 to $\mathrm{Z} 2, \mathrm{Z2}$ to $\mathrm{Z3}$, and so on
(2) It is impossible for anyone to complete an infinite number of journeys in a finite amount of time

Conclusion: It is impossible for anyone to run from $Z$ to Z* $^{*}$

## A response

The response: Contra (2), one can compete an infinite number of journeys in a finite amount of time. Indeed that is exactly what happens when anything moves.

# Russell's argument in support of this response 

We can imagine someone, $X$, getting more and more skillful at a particular task T , so that:
T first takes 1 min , then $1 / 2 \mathrm{~min}$, then $1 / 4 \mathrm{~min}$, then $1 / 8 \mathrm{~min}$, and so on.
If this is possible then $X$ can complete infinitely many tasks in 2 minutes.
If this is possible then (by analogy) (2) is false.

## Thompson's reply

If Russell's case is possible then the following case is possible.

Thompson's lamp: $X$ can put a lamp on in 1 min , then put it off in $1 / 2 \mathrm{~min}$, then put it on in $1 / 4 \mathrm{~min}$, then put it off in $1 / 8 \mathrm{~min}$, and so on.

But this case leads to an absurdity, since there is no good answer to whether the lamp is on or off in exactly 2 min.
Hence, Russell's case is impossible.

## Sainsbury's response to Thompson

- Thompson's case does not involve an absurdity.
- The imagined case, as it is described, simply leaves it open whether the light is on or off after 2 min (just as it leaves open many other things, such as whether Obama is president)
- There is therefore no reason to think that either Thompson's case, or Russell's case is impossible
- Since they both seem possible, we have good reason to think that (2) is false.


## Achilles and the Tortoise

This is Zeno's most famous paradox.

The great warrior Achilles is fast.
The Tortoise is slow
They have a race, with the Tortoise having a head start.

## Achilles and the Tortoise (cont)

Achilles starts at X , and the Tortoise starts at X .
First Achilles goes to X 1 , but by the time he gets to X 1 , the tortoise is at X 2 .
Then Achilles goes to X 2 , but by the time he gets to X 2 , the tortoise is at X 3 .
Then Achilles goes to X3, but by the time he gets to X 3 , the tortoise is at X4.
And so on
Conclusion: Achilles can never catch up to the tortoise.

## Sainsbury's response

- The Achilles and Tortoise paradox is essentially the same as the Racetrack paradox
- To see this, suppose $\mathrm{X}=\mathrm{Z}, \mathrm{X} 1=\mathrm{Z1}, \mathrm{X} 2=\mathrm{Z2}$ and so on
- Then the paradoxical conclusion relies on the assumption that Achilles can't complete the infinitely many tasks of moving from Z to $\mathrm{Z1}$, moving from $Z 1$ to $Z 2$, and so on, in a finite amount of time.
- But this assumption is false!


## Sainsbury's response (cont)

- Achilles can carry out all these tasks in a finite amount of time. And when he does he will arive at $Z^{*}$ at the same time the tortoise does


## The arrow

(1) At any instant, an arrow cannot be moving, for motion takes a period of time
(2) A stretch of time is composed out of instants

Conclusion: In any stretch of time, the arrow does not move

## Aristotle's response

Aristotle's Response: Time is not composed of indivisible instants

Problem: According to our best scientific theories (General Relativity and Quantum Field Theory), time is composed of indivisible instants).

## Sainsbury's response

(1) and (2) are true.

But what is required for an arrow to move is not that it moves-at-an-instant, but that it is in different places at different times.

Hence, even though (1) and (2) are true, then conclusion is false.

## The summing points paradox (my name)

Suppose T is a table taking up a region of space $R$ having a finite non-zero volume. Call the points that make up $R$ the R-points.
(1) $R$ has a non-zero finite volume
(2) Either the R-points have 0 volume or they have a nonzero volume greater than some number $\mathrm{d}>0$
(3) If the $R$-points have 0 volume, then the volume of $R$ is also 0
(4) If the R-points all have volume greater than $d>0$ then, since there are infinitely many $R$-points, $R$ has infinite volume
Contradiction!

## A solution

Def: $x$ is an atom iff $x$ has no proper parts
The solution: Regions of space with finite volume only have a finite number of atomic parts, each of which has a non-zero volume

Problem with this solution: Given this solution, it seems that Achilles will never catch up with the Tortoise!

